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Review of Debris-Cloud Modeling Techniques

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The state of the art in debris-cloud modeling is examined. The simulation of the debris-generating breakup event is discussed, including how the distributions of fragment number and velocity can be utilized in a parametric cloud model. The various methods available for fragment, and hence cloud, propagation are described, and the different techniques employed to calculate collision probabilities for spacecraft encountering the cloud are also discussed. It is concluded that the method of probabilistic continuum dynamics, which inherently couples cloud evolution and collision hazard assessment, offers the greatest combination of versatility and simulation accuracy among the models developed to date.

Nomenclature

A = collision cross-sectional area, km^2
a = orbit semimajor axis, km
e = orbit eccentricity
f_v = distribution of fragment velocities, (km/s)$^{-3}$
J = Jacobian of the transformation from $\dot{\boldsymbol{r}}_0$ to $\boldsymbol{r}$, s^3
M = linearized state transition matrix $\partial \boldsymbol{r}/\partial \dot{\boldsymbol{r}}_0$, s
N = second-order correction matrix, s^2/km
P_c = collision probability
$\boldsymbol{q}$ = vector of second-order velocity products, km^2/s^2
R_v = spread-velocity space of arbitrary shape, km/s^3
$\boldsymbol{r}$ = fragment position vector, km
$\dot{\boldsymbol{r}}$ = fragment velocity vector, km/s
t = time, s
V = cloud volume, km^3
V_v = volume of spread velocity space, (km/s)^3
v = relative debris velocity, km/s
μ = Earth gravitational constant, 398,600 km^3/s^2
ρ = debris density, km^{-3}
ϕ = state transition matrix $\partial \boldsymbol{r}/\partial \dot{\boldsymbol{r}}_0$, s
Ω = right ascension of the ascending node, deg
ω = argument of perigee, deg

Subscripts

B = Barrows model
C = Chobotov model
Sp = Spencer model
v = spread-velocity space
0 = value at $t = 0$

Introduction

SINCE the Space Age began, man has continued to pollute the orbital environment. Space debris now represents a significant hazard to future space operations, with around 94% of the more than 7600 cataloged objects in orbit being debris.[1] The actual number of debris objects in orbit is believed to be several times the officially cataloged population. The bulk of the debris mass is contained in spent rocket stages and inactive payloads, but the greatest danger to orbiting spacecraft comes from objects in the millimeter-to-centimeter size range. These objects are both numerous and large enough to be able to penetrate all but the most heavily shielded space structures. The most common source of such particles is on-orbit fragmentation events, and so the modeling of such events is important when trying to determine the risk they pose to current and future space missions.

Modeling the evolution of a space debris cloud and the collision risk associated with it is essentially a two-stage process. First, there is the necessity to simulate the fragmentation event itself. This takes the form of a quantitative description of the event with regard to the distributions of fragments produced and the processing of these distributions to yield a set of parameters that describe the breakup in a form that can serve as input to the second stage of the overall simulation. On receipt of the output from the fragmentation model, the debris-cloud propagator can then evolve the cloud forward in time, and the desired analysis of the spread of fragments produced can be performed. This analysis may take the form of an investigation into the size, shape, and general behavior of the cloud itself, or alternatively may concentrate on the cloud's interaction and possible collision with other orbiting objects.

This paper examines the state of the art in debris-cloud modeling, looking at both the simulation of the breakup event and the subsequent evolution of the fragments produced. The different techniques employed to calculate collision probabilities for spacecraft encountering the cloud are also discussed.

Modeling of the Fragmentation Event

Types of Breakup Model

In general, breakup models can be organized into three families—empirical, semianalytic, and complex. Empirical models are primarily derived from the curve fitting of data from impact/explosion experiments with some incorporation of analytic expressions. They tend to be based on limited data and are only tenuously based on fundamental physics. They are normally the simplest models to use, requiring only a few input parameters and negligible computation time, but they tend to be the least accurate and should only be used in context and with strict attention payed to the guidelines/caveats associated with them. Semianalytic (or semiempirical) models are developed from theoretical expressions but are normally calibrated through the use of experimental data. Hence they have a more rigorous physics base and can generally be applied to a wider range of breakup scenarios. Complex models are based on first principles of physics and include hydrodynamic (hydro) codes and structural response programs.[2]

The breakup models discussed in the remainder of this paper will all be of the first of the above three types, i.e., empirical. As stated above, these are by far the simplest to use and implement

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computationally, and because of their rapid execution are the most widely used for both debris-cloud and environment modeling.

Fragment Distributions

The cloud of debris produced by a breakup event constitutes the initial conditions (IC) for both short- and long-term hazard assessments. The key parameters of the breakup IC are velocity, mass, number, and ballistic coefficient (BC) distributions. The velocity distribution of the cloud determines its time–spatial evolution, whereas the mass relationship prescribes the lethality of a future impact, i.e., the degree of damage a fragment is likely to cause. The BC distribution has a secondary effect on both the cloud's evolution under the influence of atmospheric drag and the lethality of the fragments.[3]

On-orbit fragmentation events fall into two main categories, explosions and hypervelocity collisions, both of which contain a number of possible scenarios. Historically, explosive breakups have been the dominant source of fragmentation debris. Explosions can be physical (e.g., a pressure burst), chemical (e.g., battery- or propellant-related), or even nuclear.[4] Also, a chemical explosion may be of high or low intensity, depending upon how the explosive charge is coupled to the spacecraft structure.[5] As the physical processes involved in explosions and hypervelocity collisions are so different, each must be modeled separately regarding the ejection velocities and numbers of fragments produced. Relating mass and BC to fragment size can be considered to be a non-event-specific task, however, and so common models may be used.

In many respects, deriving and indeed using empirical breakup models is still very much a black art: models based on scant, and in some cases inappropriate, data are often employed for a much wider range of fragmentation scenarios than they were originally intended for. No universal, consistent set of fragmentation equations exists, although at least one attempt has been made to bring together a combination of empirical equations and observational data to produce a unified breakup model.[6] Whether a single set of simple equations will ever be able to accurately describe the complete range of potential breakup scenarios is doubtful, but more dedicated experimental data are certainly needed to remove many of the large uncertainties in current models.

Generation of the Debris Cloud

Generally, fragment distributions are of no real use, in the form that they are quoted, for the systematic analysis of debris-cloud propagation or collision hazards. Firstly, each distribution is continuous. These continuous distributions must be quantized to produce a set of discrete fragments. Also, the velocity models are really only speed distributions, i.e., no distribution of ejection directions is specified. The fragment distributions thus require a degree of processing before they can be used as input to a debris-cloud or environment model.

The simplest way of modeling a debris cloud is to assume that the fragments are ejected isotropically from the breakup point. The shape of the cloud is therefore initially spherical. By setting maximum and minimum debris size limits, the number of fragments produced by the breakup can be determined by using an appropriate cumulative number distribution.[2,3] From the smallest fragment size (and the velocity spreading function used, e.g., the NASA triangular[7]), the maximum debris ejection velocity or Δv can be determined. This essentially scales the cloud. A mean value of the debris density at a given time can be calculated by dividing the number of fragments in the cloud by the cloud's volume. To model a cloud with variable density and a variable growth rate, a number of concentric cloud shells or Δv bins can be employed. For example, three such shells can be used to represent the trackable, nontrackable but potentially lethal, and nontrackable but shieldable fragment categories.[8,9] The cloud's fragment-density–Δv profile will depend upon the number of shells used, their spacing, and, of course, the type of event in question. Debris densities are usually found to be highest near the cloud's centroid, and decrease rapidly towards the outer extremities of the cloud (volume dilution effect).[10,11] Certain model combinations (e.g., large numbers of small fragments with low maximum ejection velocities) and Δv binning strategies can lead to debris densities being highest at the cloud's extremities, however.[12] Typically, on-orbit explosions and collision-induced breakups will produce quite different debris clouds. This is because of the difference in fragmentation energies and mechanisms associated with the two types of event.[4] Collision-induced events generate enormous quantities of very small, fast-moving fragments and hence produce very large debris clouds. In contrast, explosions tend to produce mostly large fragments with generally lower Δv. So, although the chance of collision with some fragment is generally higher for a collision-induced debris cloud, the risk of a collision with a large (and hence potentially damaging) fragment may actually be greater for an explosive breakup. Highly energetic breakups are also not necessarily the most hazardous. More satellites may encounter such a cloud than for a low-energy fragmentation, because of the cloud's size, but lower debris densities will reduce the collision risk in each case.[13]

The type of model outlined above considers the debris cloud as a continuum, a three-dimensional envelope in space inside which all the debris is contained. No details on individual fragments are provided. Debris density is the primary cloud descriptor. This type of representation is best suited to collision hazard analysis, which uses spatial density to calculate values of collision probability. For cloud propagation only the envelope boundaries are evolved and the internal cloud structure is smoothed though the use of mean or representative fragment Δv and BC. Anisotropy and information on individual pieces of debris (e.g., orbital parameters, orbital lifetime) can only be introduced through the creation of actual fragments from the breakup model. This requires some degree of random selection of mass, Δv, etc., from the fragment distribution spreads, and also a method of assigning ejection velocity directions. All directions can be selected completely at random or can follow a direction–antidirection coupling so as to conserve angular momentum.[8] Using the magnitude of the fragment Δv, its direction, and the orbital position and velocity vectors of the parent object at breakup, the orbit of the fragment can be determined. The orbits of the individual fragments (and hence the whole debris cloud) can then be evolved using a standard orbit propagator. This approach is particularly suited to assessments of the cloud's long-term effects on the environment and the integration of fragmentation debris into a considered orbital object population. The effects of the cloud's internal structure on its dynamics and growth can also be investigated. As each fragment orbit is processed individually, a lower limit on debris size has to be set for computational practicality. This limit can, for example, be taken to correspond with the minimum trackable debris size or alternatively a spacecraft shielding threshold. Smaller particles need not be ignored, however, but can be considered on a macro level. A typical macrofragment is taken as being representative of a number of particles of a given size.

Cloud Propagation and Collision Hazard Analysis

Introduction

Modeling the evolution of a space debris cloud has been the subject of a considerable amount of research over the last few years. The quest for a computationally fast model has always forced a number of simplifying assumptions to be made to make the problem manageable. Such approximations are almost always made at the expense of accuracy, however. Many of these simplifying assumptions have been removed in the most recent models, enabling more accurate, and more realistic, simulations to be performed.

Cloud evolution models can generally be classified with respect to the length of time after breakup that they are valid for, or the period of the cloud's lifetime they aim to simulate. Within the orbital-debris community, long term usually means the period after the right ascensions of ascending nodes and argument of perigees of the fragments have become randomly distributed. It usually takes several years to reach this state. Medium term means the phase when the true anomaly has become random but the distributions of Ω and ω are still biased by the original fragment breakup values. The phase before the true anomaly has become random can be defined as the short term. The majority of the models discussed in this paper are concerned with the short-term phase.

The main purpose in accurately modeling the evolution of a debris cloud is to be able to investigate its interaction with other orbiting objects and ultimately calculate the probability that the target object will collide with a piece of debris from the fragmentation. Many of the techniques discussed next are purely propagation methods and offer no inherent facility for cloud–target encounter detection or collision probability calculation. Crude approximations or complicated geometrical calculations are then required to determine the volume of the cloud and whether or not the target is inside it at any given time. Other methods, most notably probabilistic continuum dynamics, have a direct coupling between their method of cloud propagation and collision probability calculation. Thus, the two topics can be considered to be interdependent and so will be discussed together here.

The equation generally used in the literature for calculating the collision probability over time t is

$$P_c = \rho A v t \tag{1}$$

This equation is derived from the kinetic theory of gases and Poisson statistics.[5] It assumes that the debris behaves like a rarefied gas and that P_c is very small (i.e., the probability of more than one collision is negligible).

Linearized State-Transition-Matrix Methods

Undoubtedly the simplest short-term model is that pioneered by Chobotov[8,9] and based on the Clohessy–Wiltshire (CW) rendezvous equations.[14] The CW equations are the linearized equations of relative motion between an object, in this case a piece of debris, and a given circular reference orbit (taken as being the orbit of the breakup object before breakup). They can be expressed in matrix form as follows:

$$\boldsymbol{r} = \boldsymbol{M}\dot{\boldsymbol{r}}_0 \tag{2}$$

The position and velocity vectors are both given relative to an orthogonal orbiting reference frame, the origin of which is located at the system's center of mass (c.m.). The frame's orientation with respect to the geocentric inertial frame chosen is dictated by the c.m.'s orbit radius and angular momentum vectors. The state transition matrix $\boldsymbol{M}$ can also be used to calculate the cloud volume. The calculation of the cloud volume is discussed in more detail later in the paper.

Using the above model, the cloud shape formed in relative coordinates is that of a pulsating ellipsoid, which stretches out along the c.m. orbit path on account of the different orbital periods of the fragments generated. Pinch locations are formed at integer multiples of the full and half revolution points. At these points, the cloud's volume collapses to zero. A pinch point occurs at the location of the fragmentation because, in the absence of perturbations, all debris must pass through this point, although not at the same time. Similarly, a pinch line occurs along a radial in the satellite orbit plane 180 deg from the pinch point because all debris must pass through the orbit plane along this line.

These pinch zones are termed stationary, as they are fixed inertially by the model's equations of motion. Jenkin[15] examines the phenomenon of debris-cloud pinch zones in more detail and shows that a nonstationary pinch zone exists. Identifying and locating pinch zones is important, as these are regions of high debris density and as such pose a significant threat of collision to orbiting spacecraft passing through them.

The Chobotov model is very quick and easy to use but is somewhat restricted in its application because it was formulated for circular orbits. In reality, of course, orbits are never truly circular. In fact, the circular orbit model can be used for orbital eccentricities of up to about 0.05, at which point the effects of eccentricity can no longer be neglected. To extend the range of the above model's capability, Spencer[16] uses the expressions derived by Anthony and Sasaki[17] to incorporate the effects of low eccentricities ($e < 0.25$) through the differential addition of linear eccentricity terms to the circular case solutions:

$$\boldsymbol{r} = \boldsymbol{r}_{\mathrm{C}} + \delta \boldsymbol{r}_{\mathrm{Sp}} = \boldsymbol{M}_{\mathrm{Sp}}\dot{\boldsymbol{r}}_0 \tag{3}$$

In Chobotov et al.,[8] Spencer also introduces perturbation effects (J_2 and atmospheric drag) into the simple circular form of the Chobotov model by perturbing the elements of the transition matrix $\boldsymbol{M}$. This allows estimates of the cloud's medium- to long-term characteristics to be made and also succeeds in removing the troublesome cloud-volume singularities incurred by the Chobotov model at integer half-revolution multiples after the breakup. However, the justification for adding perturbation effects to a model that is only really valid beyond the first revolution after breakup for small Δv (i.e., tens of meters per second or less) does seem somewhat questionable.

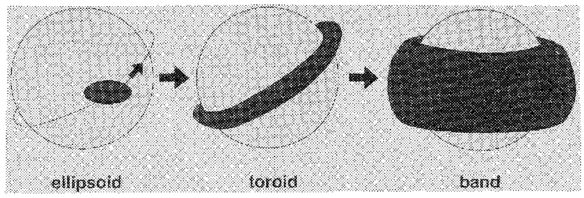

Fig. 1 Three-phase cloud model.

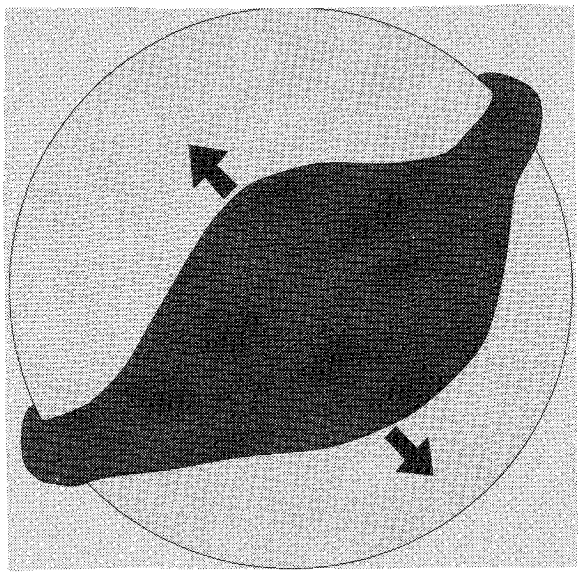

Fig. 2 Opening toroid.

Multiphase Models

Chobotov's model, with or without Spencer's modifications, can consider only the short-term evolution of a debris cloud and so on its own is inadequate for most cases. It can, however, be used as the first phase in a multiphase model. McKnight[18] considers the collision risk posed by a debris cloud in three distinct phases of its growth: ellipsoid, toroid, and band (Fig. 1). Collision probabilities are calculated for each phase separately and summed to yield the overall collision risk posed by the cloud over the time span of its complete diffusion into an indistinguishable part of the background state. The method is quite crude, however, especially in the criterion used to trigger transition between the different phases.

Jehn[19] adopts a similar approach to McKnight, but introduces an extra phase between the toroid and the band. The time taken for the effects of the Earth's asphericity to dismantle the toroid to form a band will generally be of the order of several years, and so Jehn uses an opening toroid as a bridging phase to model this gradual transition (Fig. 2).

Toroidal Models

Program DEBRIS was developed by The Aerospace Corporation to model short-term debris-cloud dynamics and to compute the probability of collision between satellites of concern and fragments in orbiting debris clouds. Early releases of DEBRIS[8,9] (versions 1.0 and 1.1) used a superposition of overlapping constant-density toroidal subclouds to model a cloud of variable density. Entry and exit times were computed for objects passing through the cloud from the geometry of the cloud structure and the orientation of the encounter. This model produced conservative but reasonable results for breakups with small maximum spread velocities. The linearizations used to describe the debris-cloud motion, and hence the dimensions of the toroidal cloud sections, meant, however, that the model became increasingly invalid as the maximum spread velocities were increased.

Frye[20] also uses a toroidal model to represent a debris cloud. The problem of determining collision probabilities for objects passing through the cloud is reduced to a two-dimensional consideration of objects passing through the debris-cloud midplane. The area-rule approach requires the complex geometric computation of target entry and exit times for passages through the cloud, but an example in the paper shows the method to be computationally rapid. The approximation of the cloud to that of a section of the debris-bounding torus does, of course, overestimate the cloud volume, as is stated in the paper. The claim that this leads to a conservative approach (by registering more encounters) seems somewhat dubious, however, as the underestimates of debris density made by this approach are likely to dominate over the contribution of any extra cloud encounters.

Crowther[21] develops an analytical model to describe the short-term evolution of a debris cloud in a circular orbit. The evolving cloud is treated as two limbs, one that advances ahead of the parent satellite locus (orbital energy less than the parent's) and one that retreats behind it (orbital energy greater than the parent's). The model represents the pulsation of the debris cloud without the volume singularities of the Chobotov model. It predicts that the volume of the retreating limb will generally be larger than that of the advancing limb and that the maxima and minima of the cloud's total volume occur at different times from the pinch locations of the Chobotov model. The limbs in the model make it relatively easy to determine when the cloud intersects with the orbits of other spacecraft, but its use of toroidal (albeit half) cloud sections tends to make it overestimate cloud volume and hence underestimate the debris density and thus collision probabilities.

Nonlinear State-Transition-Matrix-Methods

An alternative to using a multiphase or second-phase model is to extend the validity range of a linear transition matrix method through a nonlinear extension.[22] Spencer's eccentricity-corrected transition matrix model is further improved by the incorporation of the effects of second-order relative-distance terms. Again using the method described in Anthony and Sasaki,[17] a further differential correction is added, this time to the linear eccentricity-corrected solution:

$$\boldsymbol{r} = \boldsymbol{r}_{\mathrm{Sp}} + \delta \boldsymbol{r}_{\mathrm{B}} = \boldsymbol{M}_{\mathrm{Sp}} \dot{\boldsymbol{r}}_0 + \boldsymbol{N} \boldsymbol{q}_0 \tag{4}$$

The second-order corrected model is an improvement over the original linear model in that it produces a cloud that can curve around the breakup c.m. orbit. The linear model produces a cloud shape that, as it grows along track, becomes tangential to the instantaneous orbital direction of motion of the system c.m. Therefore, whereas the linear model becomes grossly inaccurate within the first half revolution after breakup for all but the slowest of cloud dispersions, the second-order model can generally be used for several hours after the event.

The second-order model above represents the useful limit of extension of the original linearized Chobotov model. The formulation of Barrows' model is considerably more complex than the simple linearized case, and although more accurate results are produced, it is debatable whether the increase in accuracy (and hence increase in length of simulations possible) is sufficient to justify the added complexity of implementation. The use of the second-order correction matrix also means that the cloud volume cannot be calculated directly from the determinant of the state transition matrix.

Another such nonlinear model is that developed by Hujsak,[23] who uses Arsenault's[24] formulation of the state transition matrix in equinoctial orbital elements. The effects of secular J_2 perturbations are incorporated, and, although the model is only formulated for circular orbits, a considerable improvement over linear models is again observed. Here, however, in contrast to Barrows' model above, the use of a single state transition matrix does enable the direct calculation of the cloud volume.

Keplerian Propagation

Nonlinear corrections can be used to extend the useful lifetime of linear models, but only up to a point. Such a model will eventually break down after a number of orbits, and the improved performance is offset by an increase in model complexity. If a longer-term analysis is required or the eccentricities of the orbits being considered are large (e.g., for geostationary transfer orbit or Molniya orbits), then the full equations of motion must be used, i.e., Keplerian propagation in the ideal, two-body case. In contrast to the state transition matrix methods described previously, Keplerian propagation deals with individual orbits in the geocentric inertial frame.

Jenkin and Sorge[25] use Keplerian propagation to investigate the effects of eccentricity on the growth and general behavior of debris clouds. The behavior of a cloud resulting from a breakup in an eccentric orbit is far more complex than that of the circular case. The variable orbit tangential velocity significantly complicates the cloud dynamics and causes the behavior of the cloud shape to be highly dependent upon the orbital position of the breakup with respect to perigee. Large along-track density variations are also observed purely as a result of the variable orbit rate. For highly elliptic orbits, these variations can be as large as two orders of magnitude.

Probabilistic Continuum Dynamics

In some of the earliest work on the subject, several authors employed the methods of statistical mechanics to examine the evolution and the spatial density of a debris cloud, and even to attempt to determine its breakup origin. Both Dasenbrock et al.[26] and Heard[27] consider the cloud as an ensemble of noninteracting particles, and although both initially embark on completely general solutions, both choose to concentrate on the simplest of cases, that of slow dispersion from an initially circular orbit. This, in fact, results in an alternative derivation of the CW rendezvous equations and a model that is essentially identical to that of Chobotov first discussed. It is the use of probabilistic continuum dynamics, however, that has been adopted by a number of authors in the last few years to produce more accurate debris-cloud models. In the probabilistic continuum method, debris-cloud motion is treated as a time-dependent mapping from spread-velocity space at the time of breakup to position space at the time of interest. The spatial density of debris at any given position can then be obtained using the Jacobian that relates spread-velocity space to position space.[28] The determination of the point at which to evaluate the Jacobian involves, for Keplerian motion, solution of the classical Gauss–Lambert problem (the problem of orbit determination given two positions and a transfer time). This can be taken to correspond to the calculation of a fragment's initial velocity vector from its position vector at a given, later time. In practice, there can be more than one such velocity vector that satisfies a given Gauss–Lambert problem. The total debris density is obtained by summing the contributions from each solution.

For example,[12] consider a breakup at time $t = 0$ and position $\boldsymbol{r}_0$. The spatial density of debris at a particular position $\boldsymbol{r}$ at time t can be found as follows. If $\dot{\boldsymbol{r}}_0$ represents an initial velocity vector that enables a fragment to perform the orbital transfer $\boldsymbol{r}_0$ to $\boldsymbol{r}$ in time t, then the number of fragments contained in a small volume element $\mathrm{d}\boldsymbol{r}$ at time t will be equal to the number of fragments that had ejection velocities in a corresponding velocity element $\mathrm{d}\dot{\boldsymbol{r}}_0$. That is,

$$\rho(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} = f_v(\dot{\boldsymbol{r}}_0)\,\mathrm{d}\dot{\boldsymbol{r}}_0 \tag{5}$$

where f_v, the distribution of initial velocities, is defined so that $f_v(\dot{\boldsymbol{r}}_0)\,\mathrm{d}\dot{\boldsymbol{r}}_0$ is the number of fragments with initial velocities in the element $\mathrm{d}\dot{\boldsymbol{r}}_0$, i.e., f_v is the spatial density of debris in spread-velocity space. The elements $\mathrm{d}\boldsymbol{r}$ and $\mathrm{d}\dot{\boldsymbol{r}}_0$ are related by

$$\mathrm{d}\boldsymbol{r} = |J|\,\mathrm{d}\dot{\boldsymbol{r}}_0 \tag{6}$$

where J can also be defined as the determinant of the state transition matrix $\boldsymbol{\Phi} = \partial\boldsymbol{r}/\partial\dot{\boldsymbol{r}}_0$. The debris density at $\boldsymbol{r}$ due to $\dot{\boldsymbol{r}}_0$ is then given by

$$\rho(\boldsymbol{r}) = (1/|J|)\,f_v(\dot{\boldsymbol{r}}_0[\boldsymbol{r}]) \tag{7}$$

It should be noted that the state-transition-matrix methods described earlier and the probabilistic continuum-dynamics method are conceptually identical. For example, $\boldsymbol{M}$, used in the Chobotov model described previously, is merely a linearized version of $\boldsymbol{\Phi}$ and is formulated in relative, as opposed to geocentric, coordinates. A distinction is made between the two, however, since the former implementations of the method dealt purely with cloud evolution and simple estimates of cloud volume, while the latter has primarily been utilized as a tool for debris-density and hence collision-probability calculation, with debris propagation carried out only implicitly.

Cloud volume can be calculated in a similar way to spatial density. The above approach, although completely general for any type of fragment trajectory and spatial distribution of spread-velocities, is in fact only valid for small regions of spread-velocity space and hence position space. In such small regions, the transition matrix can be assumed to be constant. When the entire cloud is small, then the volume of the whole cloud can be evaluated using this technique, i.e.,

$$V = |J| V_v \tag{8}$$

Hence, in the case of an isotropic breakup, the volume of the debris cloud in position space at a given time is the multiple of the relevant Jacobian and the volume of the debris-bounding spread-velocity sphere. Here the Jacobian is referenced to the center of spread-velocity space, i.e., the state of the breakup object at breakup. When the cloud increases in size, however, the approximation that the state transition matrix is constant throughout the cloud cannot be used, and the cloud volume must be obtained by integrating over all of spread-velocity space, i.e.,

$$V = \int_{R_v} |J| \, \mathrm{d}V_v \tag{9}$$

Hujsak[23] proposed that his nonlinear transition-matrix model could be used to investigate the variation of spatial density within a debris cloud. By using the inverse of the state transition matrix, an approximate solution to the Gauss–Lambert problem could be found. Hujsak[29] then went on to attempt to implement Heard's full nonlinear solution using Goodyear's[30] formulation of the state transition matrix and Gooding's[31] solution to the Gauss–Lambert problem. The original version of Hujsak's paper,[29] however, contained a number of fundamental mistakes, and a bug in his computer code caused errors in his debris-density maps.

Housen[12] tackled the same problem but with a greater degree of success. He also used Goodyear's[30] algorithm in the calculation of the Jacobian, but chose the algorithm of Sun et al.[32] to solve the Gauss–Lambert problem. His analysis revealed order-of-magnitude density variations throughout the cloud, with the highest densities occurring, as expected, near the pinch locations. For the low-velocity breakup model used, he showed that the cloud density peaks produced can exceed the background level by up to two orders of magnitude.

A debris-density calculation algorithm like that outlined above can also be used in the calculation of collision probabilities for a target object that passes through the debris cloud. Interactions with the debris cloud formed by a fragmentation event produce spikes in the target object's overall (i.e., background debris environment plus cloud) curve of instantaneous collision probability vs time. For an analysis that focuses on the collision risk to a target object due to a debris cloud rather than on the cloud itself, values of debris density need only be calculated at positions along the target's orbit. If, for a given time step, the target object is physically outside the bounds of the debris cloud, then the spatial density of debris will be zero. If it lies inside the cloud, then a nonzero value will be returned. Hence, not only is the value of the debris density determined for subsequent calculation of the collision probability at each point, but an implicit and accurate detection method is established for determining if and when any encounters between the target and the debris cloud actually occur. The technique can be applied to any scenario and can even employ a completely arbitrary breakup model (i.e., nonisotropic ejection of fragments).

A generalized (i.e., numerical) implementation of the above method is employed in The Aerospace Corporation's program DEBRIS, version 3.1.[10] In DEBRIS3.1, the two-point boundary-value problem (the Gauss–Lambert problem in two-body dynamics) is solved using a generalized vector root-solving algorithm, and the state transition matrix is calculated numerically by determining the perturbations in position space that result from small perturbations in spread-velocity space. In Jenkin,[10] two numerical examples (two target scenarios with a common breakup event and hence debris cloud) are used to show the program in operation. In the first example, the target satellite passes close to the debris cloud's first whole-revolution pinch point (i.e., the breakup location). The collision probabilities calculated are found to increase by several orders of magnitude as the target passes close to the breakup point. Jenkin et al.[33] describes the real-time collision hazard assessment performed for shuttle mission STS-60 and the Russian space station Mir, which were both in orbit at the time of the Clementine–Titan II upper-stage breakup (Feb. 7, 1994) and hence were potentially in danger as a result. The breakup time and location were estimated, and initial results of the collision hazard analysis were obtained while Discovery was still in orbit. The collision risks to both the shuttle and Mir were found to be minimal and insensitive to the determination of the breakup. This example is a perfect illustration that short-term collision hazard assessments can be used in actual operational situations and act as a valuable and indeed necessary complement to the close-approach algorithms that look for conjunctions with the trackable population.

Barrows et al.[34] outlines the theory behind, and the development of, the analytical debris-cloud collision hazard assessment program TARGET3.0. The method of universal variables[35] is used for the solution of the Gauss–Lambert problem, and Goodyear's[30] algorithm is employed for evaluation of the Jacobian. Also included in TARGET3.0 is a simple, nondirectional model of the low Earth orbit debris environment[36] for comparison of cloud-related collision probability values with those predicted from the background population. A case study is performed using the polar platform ENVISAT-1 as the target object, and the effects of a variety of breakup scenarios and locations are examined. Instantaneous values of the collision probability up to 5 orders of magnitude greater than the background are registered for certain breakup scenarios. TARGET has also been used to investigate the effects of fragmentations on satellite constellations. In Barrows et al.,[37] TARGET3.0 is used in batch mode to assess the risk of a cascade fragmentation occurring within the proposed Iridium constellation, following the breakup of either one of the constellation satellites or a constellation launch vehicle. The collision-induced breakup of one of the constellation satellites is found to be the most hazardous scenario for the remainder of the constellation, but the values of collision probability recorded are sufficiently low to mean that the likelihood of a cascade occurring is extremely remote. In Crowther et al.,[38] TARGET4.0 is used assess the short-term collision risk to an 800-satellite constellation following the fragmentation of one of the member satellites because of a debris collision. The short-term collision risk to each remaining satellite in the constellation is found to be highly dependent on its phasing and position in the constellation relative to the fragmented satellite. The likelihood of a cascade fragmentation resulting is again found to be low.

Barrows et al.[39] use the numerical examples in Jenkin[10] for a comparison of programs DEBRIS3.1 and TARGET3.1 (with respective fragmentation models IMPACT3.0 and BREAKUP3.0). It is found that while DEBRIS3.1 and TARGET3.1 are in good general agreement when used with a common fragmentation model, when IMPACT3.0 and BREAKUP3.0 are used to provide input to their respective partners, significant differences are observed in the results produced. These discrepancies serve to illustrate the importance of accurately simulating the fragmentation event for the results obtained from debris-cloud models.

Summary

A critical review of the major developments made in debris-cloud modeling has been presented. The treatment of debris-cloud collision risk analysis as a form of the classical Gauss–Lambert problem has enabled significant improvements to be made in simulation software in recent years, enabling many of the assumptions traditionally used in such analyses to be removed (e.g., linearized motion, mean values of debris density, and isotropic fragment ejections). A good level of agreement in results can now be achieved between simulation codes, but more work is still needed, especially in the area of breakup modeling.

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